Experiment Nr. 24

Josephson effect

1. General information

1.1 Introduction

The Josephson effect is an example of a macroscopic quantum phenomenon. There are several important applications, which utilize the Josephson effect. Quantum interference in the ring structures with two Josephson contacts are used for measurements of magnetic flux with accuracy better than one flux quantum. Such devices are called superconducting quantum interference devices (SQUIDs) and widely used in science and engineering. The Josephson effect is used in a very precise and stable voltage standard and in various switches and detectors.

In this experiment we will study several properties of Josephson contacts, which are important for understanding devices utilizing the Josephson effect. A point contact between niobium wires will be fabricated. By measuring the properties of this contact the DC and AC Josephson effects will be investigated including current-voltage characteristics and microwave radiation.

1.2 Literature

[1] Description of the experiment

1.3 Preparation

The following topics have to be learned:

- Critical temperature $T_c$
- Cooper-pairs
2. Preparation

Answer the following questions in written form before you start the experiment:

1. Why the 4-point method of resistance measurements has to be used?

2. At which voltage are you expecting the radiation emission from the Josephson contact at the frequency of 10 GHz? At which voltage are you expecting the second harmonic for the same emission frequency?

3. Deduce the differential equation for $\gamma$ in RSJ model and the Josephson equations (see the file for preparation).

4. Which analogy exists between the parameters $C$, $R_n$, $\gamma$, $I$ of a Josephson contact and the parameters of a physical pendulum?

5. Show the transformation invariance of the equation 2.10 (the 2nd Josephson equation).

3. Set-up

3.1 Sample

The dipstick and the niobium contact used for this experiment is shown schematically in Figure 3. The technique of a niobium tip preparation, fixing and contacting will be described later.
3.2 Dipstick

The experiment is performed with a dipstick (Fig. 3.1), which can be cooled down to temperatures below the critical temperature of niobium (~9.3K) in a liquid helium dewar. The dipstick has a circular waveguide, which works in the region between 9.8 and 11.5 GHz. The horn antenna is connected to the lower part of the waveguide to receive the radiation from the Josephson contact, which is placed in the centre of the antenna. The dipstick is built in such a way, that the Nb-tip can be slowly brought into contact with the second flat Nb-wire.

3.3 Measurements

Current-voltage characteristics
An externally controlled current source allows to apply up to ±100 mA DC current. The current is monitored with the help of a digital voltmeter 1 (DVM1). The voltage drop between two sides of the Josephson contact is measured by the second digital voltmeter (DVM2). Note: The current is shown in units of volts, which has to be converted to Amperes according to the conversion coefficient X, where X is the current range of the current source used during measurements. The current range can be adjusted according to the requirements between $10^{-6}$ to $10^{-1}$ A. Both DVMs are connected to a computer via GPIB bus and the data from DVMs are acquired with the help of kgPlot program.

All the wirings have low-pass filters to short high-frequency stray signals, e.g. noise.

![Diagram of the current- and voltage measurements](image)

**Fig. 3.3: Diagram of the current- and voltage measurements**

**Radiometer (Fig. 3.4)**

The radiation of the Josephson contact with frequency $f_{JK}$ is picked up by the horn antenna and transferred to a low-noise block downconverter (LNB) through the waveguide as it is shown in Fig. 3.4. The low-noise block downconverter is shown schematically in Fig. 3.5. The signal passes through the high-frequency amplifier, and then mixed with the signal from the internal local oscillator. The resulting signal with an intermediate frequency $f_{ZF}=f_{JK}-f_{LO}$ is transferred to a radiometer. In our experimental set-up the frequency of the local oscillator $f_{LO}$ is around 10 GHz.
With the tuner one can choose the intermediate frequency between 900 MHz and 1759 MHz in a window 30 MHz or 70 MHz, respectively.

With 50 Ω resistor, the 70 MHz signal is not reflected and the high-frequency voltage on the resistor is rectified by a diode. After filtering with a low-pass filter the voltage proportional to the root of the radiation power is measured with the third digital voltmeter (DVM3).

**Temperature measurement**
The platinum resistor PT100 is fixed near the sample and used for temperature measurements. With the help of the calibration table the measured resistance of the PT100 can be converted to Kelvin.

4. Experiment

4.1 Cooling down the dipstick

The adviser should be present during mounting the dipstick onto a helium dewar.

Rotate the screw driver at the top of the dipstick anti-clockwise till it stops. It separates the Nb-tip and the flat Nb-wire. The precise position for the Nb-contact would be given by your adviser. Open the Helium dewar, mount the dipstick on the top of the dewar and fix it with a clamp. To cool the sample down move the dipstick slowly inside the dewar. Hold the cooling speed lower than 20 K/min. Reach the temperature below the critical temperature of niobium (9.3K).

4.2 Experimental procedure

The experiment consists of three parts.

a) Measure a hysteretic current-voltage characteristics (I-V) of the Josephson contact (corresponds to the underdamped state).

b) Measure a non-hysteretic I-V which corresponds to the overdamped state.

In order to change from the underdamped to overdamped state, adjust either the distance between Nb wires or the temperature.

c) Measure an electromagnetic emission using the contact in the overdamped state

For the emission measurements use the radiometer. Try to find the 1st and the 2nd harmonics

4.3 Warming up the dipstick.

Pull the dipstick up slowly keeping the warming speed lower than 20 K/min. When the temperature is close to the room temperature, remove the dipstick from the Helium dewar. Close the dewar. This must be done with your adviser

5. Analysis
1. Discuss your different I-Vs measured during the experiment and find from the slope at high bias voltages the normal resistance \( R_n \).

2. Estimate the McCumber-parameter \( \beta_c \) using the hysteretic I-V with the help of Equation 2.16. Calculate the self-capacitance of the Josephson contact using the obtained value of \( \beta_c \) (see 2.15).

3. Find the critical current \( I_c \) and the differential resistance \( \frac{dU}{dl} \) at voltages of the emission maxima from the current-voltage characteristic. How does the differential resistance affect the emission line width.

4. Calculate the value of \( 2e/h \) from the position of the emission maxima using the Equation 1.9. How big is the deviation from the literature value?
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1. Safety instructions

Students are permitted to perform the experiment only together with the tutor or after being instructed by your supervisor about possible threats.

1.1 Liquid Helium handling

General

Liquid Helium is recycled in order to reduce operating costs. Storage vessels (helium dewars) should be therefore always connected to the return gas line. A short transport within the building is allowed. Tilting of the helium dewars is not allowed. The transport in the lift is permitted only without accompanying persons.

Dangerous

Cold burnings and freezing could occur due to the contact with a liquid, cold gas or parts which were in contact with them (temperature of the liquid Helium is 4.2 K). There is a danger of suffocation by large quantities of out coming helium gas (due to oxygen deficiency). In the case of uncontrolled helium gas leak, everybody should immediately leave the room.

Notes on liquid helium handling

- Gloves and safety goggles are required for personal protection during mounting and dismounting the dipstick.
- Helium level in the helium dewar must be checked before the experiment
- Never leave the Helium dewar opened or without connection to the helium return line.
- Only slow cooling/warming of the dipstick is allowed. A typical cooling/warming speed should be less than 20 K/min. Fast cooling can cause an extensive volatilization of helium and, in the worst case, can damage the isolation vacuum of the dewar.
1.2 Electronic devices

Voltages above 10 V are not used in the signal lines. Therefore, the assembling of the connection lines can be changed safely. However, any changes in the experimental set-up should be done only by the adviser. Any problems with measurements should be immediately reported to your tutor.

2. Basic information

2.1 Superconductivity

Heike Kamerlingh Onnes expected disappearance of metal conductance in mercury (Hg) at liquid helium temperature, when he was cooling mercury for the first time in Leiden in 1911. His expectation was based on the assumption that the conducting electrons would have no sufficient kinetic energy to move. Contrariwise, he found that the electric conductance is getting better with cooling and sharply changes to some infinitely high value (corresponds to a zero electrical resistance) at around 4.2 K. He has discovered the superconductivity in Hg (Fig. 2.1).

![Figure 2.1: Superconductivity in mercury](image)

With development of the quantum mechanics in two decades after this discovery, it became clear that the conducting electrons, as Fermi-particles, build the gas, with a high energy of several electron-volts even at $T = 0$. 
The electric conductance is governed only by the highly energetic electrons in the conduction band at the Fermi energy, whose mobility in the perfect crystal is infinitely high. In practice, due to defects in crystals, (already by $T=0$) and also (by higher temperatures) due to scattering on phonons the value of conductance is limited. This concept explains normal metallic conductance but the explanation of the superconductivity remained mysterious for a long time, in particular, because of its independence on the purity of materials.

The superconducting state is obviously a new thermodynamic state of the electron gas, in which the electrons can move non-dissipative like in atoms or molecules. The electrons lose their entropy (no degrees of freedom anymore, highly ordered state). This explains, why the superconducting state can be stable only at low temperatures. The state is thermodynamically stable at the minimum of the free energy $F = U - T\delta$. To compete with a normal state (higher entropy) the intrinsic energy (or binding energy) $U$ in the superconducting state has to be smaller. The electrons see the whole metallic crystal as a giant molecule, in which their wave packets move absolutely phase locked. Therefore, it is reasonable to speak about the phase center of electrons. In the state with no current the phase is constant in the space. With the change of the phase in space ($\frac{\partial}{\partial x} \Psi(r) \neq 0$) the super current starts to flow. In a superconducting ring, for example, the wave train should be closed, which is equivalent to the quantization of the angular momentum:

$$\oint (\vec{p} + e\vec{A}) ds = n \cdot \hbar$$  \hspace{1cm} (2.1)

$\vec{p}$ is the mechanical momentum and $\vec{p} + e\vec{A}$ is the canonical momentum of the electrons (in presence of the magnetic field with the potential vector $\vec{A}$). From (2.1) it follows that the magnetic flux is quantized. A simple continuous superconductor (for example, a sphere), which is placed in not too strong magnetic field, prefers from energetic point of view quantum flux state with $n = 0$, in which no magnetic field $\vec{B}$ can be found inside of the superconductor. The external magnetic field is shielded by the super currents in the thin surface layer $\lambda$ ($\approx 500$ Å, London's penetration depth) Fig. 2.2.
This remarkable state is not in contradiction with the Equation (2.1) in the surface layer, as simultaneously the $\vec{B}$ field exists and the current is flowing in such a way that both terms $\vec{p}$ and $e\vec{A}$ in Eq. (2.1) cancel each other.

Other remarkable effects could be observed, when two superconductors are brought in a weak contact, e.g., tip-contact via oxide layer or a layer of a normal conductor.

![Figure 2.2: Meissner effect](image)

Supraleiter 1

$\Psi_1 = \sqrt{n_1} \exp(i\varphi_1(\vec{r}))$

Supraleiter 2

$\Psi_2 = \sqrt{n_2} \exp(i\varphi_2(\vec{r}))$

![Figure 2.3: Josephson contact](image)

Fig. 2.3 shows the principle of such contacts. The phase centre of superconducting electrons in superconductors 1 (SC1) and 2 (SC2) is well-defined. Due to the weak coupling between two superconductors it is possible to have a phase difference between 1 and 2. In the superconducting theory of Bardeen, Cooper and Schrieffer (BCS-Theory, 1957) the electrons in superconducting electron gas are correlated in pairs (Cooper-pairs). Their binding energy stabilizes the superconducting state (despite of the low entropy). A Cooper pair can be considered as a wave packet, which extends by the coherence length $\xi$ ($\xi \approx 10^4 \text{ Å}$). If the contact layer between the superconducting layers is considerably thinner ($\approx 10-10^2 \text{ Å}$), it is possible that Cooper pairs can tunnel through the non-superconducting layer without breaking. This means that the weak super
current (Josephson current) between superconductors is possible. Correlation of the Josephson currents with the phase difference between SC1 and SC2 in or without magnetic field as well as with and without electrical potential difference between superconductors is described as Josephson effects.

2.2 Josephson effects

There are some very important applications of superconducting materials, where the Josephson effects are used. E.g., Josephson contacts could be used as extreme fast circuit elements and a very precise and stable voltage standard. Quantum interference in the ring structures with two Josephson contacts is used for measurements of magnetic flux with accuracy better than one flux quantum.

In this experiment we study a niobium point contact. Different types of Josephson contacts will be achieved by controlling the distance between the superconductors and the temperature. The AC-Josephson effect will be measured via detecting microwave radiation in X-band (X-band: 8.2 - 12.4 GHz).

2.2.1 Theory

Superconductivity is a quantum phenomena on macroscopic scale. The BCS-theory (1957), describes the superconducting state as a condensate of phase-correlated bonded in pairs electrons (Cooper-pairs). The total wave function can be written as

\[ \Psi = \sqrt{n} \cdot \exp (i \varphi (r)) \tag{2.2} \]

where \( n \) is the density of Cooper-pairs and \( \varphi \) is the phase of the wave function. \( \Psi \) describes the center of mass of the superconducting electrons. The general expression for quantum-mechanical current density is

\[ \vec{j} = \frac{i \hbar}{2m^*} \cdot (\Psi^* \vec{\nabla} \Psi - \Psi \vec{\nabla}^* \Psi) - \frac{e^2}{m^* c} \cdot \vec{A} \cdot \Psi \Psi^* \tag{2.3} \]

With the space independent Cooper pairs density \( n, q = -2e \) and \( m^* = 2m \) one gets

\[ \vec{j} = -\frac{\hbar e n}{m} \cdot \vec{\nabla} \varphi - \frac{2e^2 n}{m c} \cdot \vec{A} \tag{2.4} \]

Here \( e \) is the elementary charge and \( m \) is the mass of one electron. Equation (2.4) shows the relation between the current density \( \vec{j} \) in the superconductor, the phase \( \varphi \) of the wave function and the vector potential \( \vec{A} \). From (2.4) the 2nd London equation \( \vec{\nabla} \times \vec{j} = -\frac{1}{\mu_0} \vec{B} \) can be derived, which explains the Meissner effect.
As the current density is gauge invariant, the phase of the wave function by the invariant transformation $\vec{A} \rightarrow \vec{A} + \vec{\nabla} \chi$ transforms to

$$\varphi \rightarrow \varphi - \frac{2e}{\hbar c} \chi. \quad (2.5)$$

Invariance here means that the equation is independent on the chosen vector potential $\vec{A}$ to which a gradient of an arbitrary potential function $\chi$ can be added without changing the magnetic field $\vec{H}$:

$$\vec{A}' = \vec{A} + \vec{\nabla} \chi$$

$$\vec{H} = \text{rot} \vec{A}' = \text{rot} \vec{A} + \text{rot} \vec{\nabla} \chi = \text{rot} \vec{A}. \quad (2.6)$$

### 2.2.2 Josephson equations

If two superconductors are brought into a weak contact, i.e., via the region with a considerably reduced order parameter $\Psi$ (Fig. 2.3), the whole system is still described by a uniform condensate wave function. Both uncoupled electrons and Cooper-pairs can cross the region of the weak link. The resulting effects were predicted by B. D. Josephson in 1962.

The current through the barrier depends on the phase difference $\gamma = \varphi_2 - \varphi_1$ between both sides: $j_{12} = f(\gamma)$ analogously with the correlation between the current and the phase in a bulk superconductor (see 2.4).

As a time reversal transformation changes the direction of the current and the sign of the phase, this implies that the current needs to be an odd, $2\pi$-periodic function of the phase difference. It can be shown that - under the assumption of a weak coupling (e.g. tunnel contact) - the first Fourier component of the expansion of $f$ in terms of the phase difference is sufficient, which means that $j_{12}$ is a sine of the phase difference:

$$I = I_c \sin \gamma \quad (2.7)$$

This equation is known in the literature as the 1st Josephson equation. For a tunnel contact the critical current $I_c$ at $T = 0$ is proportional to the energy gap (so called Ambegaokar-Baratoff-relation): $I_c \propto \Delta$.

This current has to be also gauge invariant as well in magnetic field. The corresponding extra term of the vector potential in the gauge invariant phase difference leads to the following equation:
By increasing the current \( I \) above the critical value \( I_c \) the current cannot be carried out by the super-current alone. The additional current of normal electrons leads to a potential difference between the superconducting sides \( U \) proportional to the normal resistance \( R_n \). To describe it quantitatively, the effect of a gauge transformation on the properties of a time-independent scalar electromagnetic potential \( \Phi \) has to be taken into account:

\[
\Phi \rightarrow \Phi - \frac{1}{c} \frac{\partial \Phi}{\partial t}
\]  

(2.9)

This equation together with Eq. 2.5 leads to

\[
\dot{\gamma} - \frac{2e}{h} \cdot U = 0
\]

(2.10)

which is called the 2nd Josephson equation.

The first Josephson equation shows the relation between the phase difference at the Josephson contact and the super-current across the contact.

The second Josephson equation shows, that the phase difference changes linearly with time if a potential difference between two superconductors is finite.

### 2.2.3 Josephson contacts
2.2.4 DC - Josephson effect

Without a potential difference between the two superconductors, as follows from Eq. 2.10,
\[ \gamma = \gamma_0 = \text{const}. \]
Therefore, a constant current \( I = I_c \cdot \sin \gamma \) flows through the contact, if \( I \) lies in the range \( \pm I_c \) (critical current).

This equation looks similar to the equation for coupled pendulum: no energy exchange occurs for in phase or out of phase oscillations. The maximum energy exchange takes place for a phase shift of \( \pm \pi/2 \).

2.2.5 AC - Josephson effect

When the voltage \( U \) between two superconductors is not zero, \( \gamma = \gamma_0 + (2e/\hbar)Ut \) can be derived from Eq. 2.10 and, consequently, from Eq. 2.7 follows:

\[ I = I_c \cdot \sin(\gamma_0 + \frac{2e}{\hbar} Ut) \]  \hspace{1cm} (2.11)

Then, an alternating current of Cooper-pairs with frequency

\[ f = \frac{2e}{\hbar} \cdot U \]  \hspace{1cm} (2.12)
flows across the contact. As \( 2e/\hbar = 483.6 \text{ MHz}/\mu\text{V} \), the frequency of electromagnetic emission lies mostly in the microwave range for experimentally accessible values of voltages \( U \). The radiation emission of the frequency \( f \) can be detected with the help of radiometer.

The AC-Josephson effect can be described as following: the tunneling Cooper-pairs in presence of applied voltage \( U \) accelerate and gain the energy \( 2eU \). As the Cooper pairs are not allowed to change their energy in tunneling process, a photon with the energy \( hf = 2eU \) (Eq. 2.12) is emitted.

2.2.6 Josephson contact in magnetic field

Taking into account the finite size of the Josephson contacts a correlation between \( I \) and the magnetic flux \( \Phi \) through the contact is derived from Eq.(2.8). It can be shown that this correlation has the form:
where $\Phi = 2\lambda_L \cdot l \cdot B$ (see, for example, Ref. [1]). The parameter $l$ is the size of the contact perpendicular to the magnetic field and $\lambda_L$ is the London penetration depth. This function is shown in Fig. 2.5. It has the same form as the diffraction image on a slit.

\[ I_c(H) = I_c(0) \cdot \left| \frac{\sin\left(\frac{\phi}{\phi_0}\right)}{\frac{\phi}{\phi_0}} \right| \quad (2.13) \]

2.3 Current-Voltage characteristics of Josephson contacts

More complicated is the description of the Josephson contact in the case of an extra current of normally conducting electrons with the resistance $R_n$ and with the capacitance $C$ between the electrodes. These two parts are taken into account in the RSJ-model (Resistively Shunted Junction). The schematic diagram of RSJ model is shown in Fig. 2.6.
Then, the total current across the contact can be written as

$$I_{\text{tot}} = I_c \cdot \sin \gamma + \frac{U}{R_x} + C \dot{U} \quad (2.14)$$

In order to obtain a current-voltage characteristic, a total current $I_{\text{tot}}$ is applied across the Josephson contact and the voltage $U$ across the contact is measured. Due to the non-linear behavior, the frequency spectrum of the current contains not only the basic frequency according to Eq. 2.12 but also higher harmonics.

Inserting the second Josephson equation (2.10) in equation (2.14) yields a non-linear differential equation for $\gamma$ that corresponds to that of a physical pendulum [4]. Solving this differential equation numerically, one has to distinguish two limits with respect to the McCumber parameter $\beta_c$

$$\beta_c = \frac{2 \pi c B R_s I_c}{\Phi_0} \quad (2.15)$$

which is the measure of the quality factor $Q$ of the resonant circuit. Here $\Phi_0$ is the flux quantum.

- $\beta_c > 1$ (underdamped)
  For $I < I_c$ there are two stable solutions. Therefore means the I-V curve is hysteretic (Fig. 2.7a). By increasing the bias current $I_{\text{tot}}$ from zero to some value, which is lower than the critical current, the tunnel current is provided by Cooper-pairs and no potential difference between the contacts can be observed. When the total current exceeds the critical current $I_c$, the contact switches immediately (within 1 ps) to some final voltage value. This property can be used in switching devices. By decreasing the bias current the return curve shows the resistive state even for currents lower than the critical current $I < I_c$. Only when the bias current becomes lower than the retrap current $I_R$ the contact switches back to its state without voltage. The McCumber-parameter can be estimated using this hysteretic curve:

$$\frac{I_R}{I_c} = \frac{4}{\pi \sqrt{\beta_c}} \quad (2.16)$$

- $\beta_c < 1$ (overdamped)
  I-V curve shows a non-hysteretic behavior (Fig. 2.7b). In the limit of a vanishing capacitance the McCumber-parameter $\beta_c = 0$ and the differential equation can be solved analytically. The time averaged voltage $\langle U(t) \rangle$ for $|I_{\text{tot}}| > |I_c|$ is equal to

$$\langle U(t) \rangle = \pm R_n \sqrt{I_{\text{tot}}^2 - I_c^2} \quad (2.17)$$
2.4 Radiation emission
The radiation power at a certain frequency depends on many parameters of the Josephson contact. Due to nonlinearity of the Josephson contact a part of the radiation power is emitted at higher harmonics.

![Figure 2.7: Typical Current-Voltage characteristic (a) of a tunnel contact and (b) of a weak link.](image)

Furthermore the radiated power is determined by the impedance matching of the Josephson junction to the characteristic impedance of the space. Fig. 2.8 shows an emission curve measured as a function of voltage on a point contact.

![Figure 2.8: I-V characteristic of a Niobium contact (solid) and radiometer signal P at frequency \( f = 12.75 \text{GHz} \) (dots).](image)

2.6 Summary of terms
• Critical temperature $T_c$
• Cooper pair
• Meissner effect, ideal diamagnetism
• London penetration depth
• Josephson contact, tunnel current
• Josephson equations
• DC- and AC-Josephson effects
• Current-voltage characteristics (I-Vs) of Josephson contacts

Bibliography